Pass the Basketball – Linear and Quadratic Data Collection Activities

Many teachers have probably seen a linear version of this activity. Students determine the time it takes for different numbers of students to pass a ball from one student to the next. If the students pass the ball at a relatively constant rate, the data collected and graphed (time versus number of students) can be modeled by a linear function. The activity can be modified to collect data that is logically modeled by a quadratic function. Questions are provided for each version of the activity. A basketball and a stopwatch are needed for both activities.

It isn't easy to find a "real world" connection for these activities. Perhaps passing a basketball quickly, as is done in the linear activity, could improve a person's eye-hand coordination.

Linear Activity:

One student will be needed to serve as the timer and another to record the data. The remaining students in the class should stand in a straight line

Number of Students	Time (seconds)
5	
10	
15	
20	
25	

When the timer says "Start," the first student in line passes the ball to the next student and this continues to the fifth student. The timer stops the stopwatch when the fifth student receives the ball. The recorder enters the time in seconds required for the ball to be passed to the fifth student. The first student in line gets the basketball back and the process is repeated for the first ten students in the line. Always start the basketball back with the first student and add five students each time. With a small class, add two students each time you repeat the data collection.

After the data are recorded, share the data with all students and ask them to answer the linear activity questions.

Quadratic Activity:

One student will be needed to serve as the timer and another to record the data. The remaining students in the class should stand in a straight line.

Number of Students	Time (seconds)
5	
10	
15	
20	
25	

In this version of the activity, the first student says "A" and then passes the ball to the second student. The second student says "A B" and then passes the ball to the third student. This pattern continues until the fifth student receives the ball and says "A B C D E." After the fifth student says the last letter (E), the timer stops the stopwatch and the time is recorded.

The first student in line gets the basketball back and the process is repeated for the first ten students in the line. The tenth student in the line will say "A B C D E F G H I J" when the ball is received and the timer stops the stopwatch after this student says "J." The time for the ten students is recorded. Repeat this process, adding five students each time, until data are collected for all students in the line.

If there are more than 26 students in the class, the next student will say all of the letters in the alphabet and then say "one." The student after this one would say all of the letters in the alphabet and then say "one two." This data collection process will take more time than the linear activity but the students enjoy it even if they get tongue-tied.

Since the second change is theoretically constant, these data can be modeled by a quadratic function. Consider students three and four in the line. The third student said three letters before passing the ball and the fourth student said four letters. They did not change the "slow down" time by the same number of letters but the change in that "slow down" change was one letter. This is the same for the ninth and tenth students in the line. The ninth student slowed down the passing of the ball by nine letters while the tenth slowed it down by ten letters. The change in the change (second change) is, again, one letter. With quadratic functions, there is a constant second change in the dependent variable values. This can be illustrated using a quadratic function such as $y = 2x^2 + 3x - 1$ with ordered pairs (1, 4) (2, 13), (3, 26), (4, 43) and (5, 64). The first changes in the dependent variable are 9, 13, 17, and 21, respectively. The second changes are 4, 4, and 4. Note that the independent variable values ({1, 2, 3, 4, 5}) in the ordered pairs increase by one unit each time.

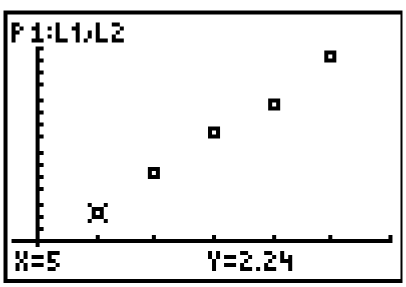
Sample Data with Solutions

Linear Activity

Number of Students	Time (seconds)
5	2.24
10	5.3
15	8.41
20	10.59
25	14.29

Questions for Linear Activity

1. Graph the data with the number of students on the x-axis and the time in seconds on the y-axis. Why, in this problem, is it logical to choose the number of students as the independent variable and the time as the dependent variable?



The time it takes to pass the ball depends on the number of students passing the ball.

2. If you're using a graphing calculator, press TRACE and move the cursor (using the arrow keys) on the graph to choose two points that you think would be located on a line of best fit. Record the ordered pair for each point. If graphing by hand, pick two points that would be located on a line of best fit.

For the sample data shown, the points chosen were (10, 5.3) and (25, 14.29).

3. Find the slope of the line that could be drawn through these two points.

The slope is approximately 0.6.

4. Write the slope as a fraction with the appropriate units.

O.6 seconds / 1 person

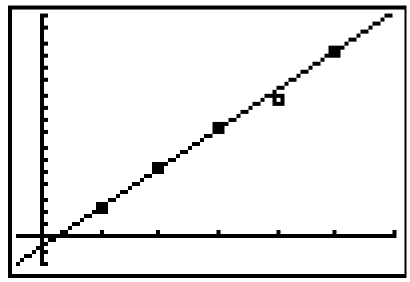
5. What does the slope represent in this problem?

If one more person is added to the line of students, it will take an approximately 0.6 of a second longer to pass the ball. You might ask the students to describe what would happen if 10 more students are added to the line. It would take approximately six more seconds to pass the ball.

6. Find the equation of a line of best fit. Write the equation of your line of best fit in the form y = m x + b. Graph this line.

5.3 = 0.6 (10) + b
b = -0.7

y = 0.6 x – 0.7



7. What is the unit for the value of b in your equation?

The unit for b is seconds.

8. What does the value of b represent in this problem? Note that b is the y- value of the y-intercept of the line.

The value of b has no real meaning in the context of this problem. It couldn't take -0.7 of a second for zero students to pass the ball. If the b value in a linear model for this activity is positive, that value could represent how long it takes the first student to start passing the ball after the timer says "Start." .

9. Find the x-intercept. What does the x-intercept represent? Does the x-intercept have a real meaning in this problem?

The x-intercept for the equation found in number 6 is approximately 1.2. This would indicate that 1.2 students take zero seconds to pass the ball. This has no real meaning in the context of this problem.

10. Give the domain and range of your equation for this problem situation.

The domain is the set of positive integers and the range is the set of positive real numbers.

11. Using your equation, predict the amount of time it would take 100 students to pass the basketball.

y = 0.6 (100) – 0.7
y = 59.3 seconds

12. Using your equation, find the number of students who could pass the basketball in two minutes.

120 seconds = 0.6 sec/person (x) – 0.7 seconds
x = 201.17 people

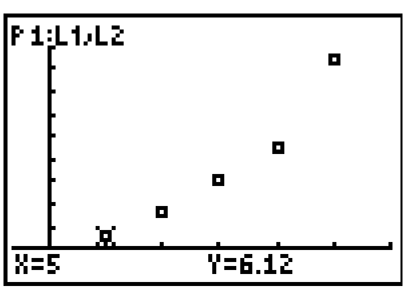
Approximately 201 people could pass the basketball in two minutes.

Quadratic Activity

Number of Students	Time (seconds)	First Change	Second Change
5	6.12		
10	16.08	9.96	
15	31.18	15.1	5.13
20	45.43	14.25	-0.85
25	84.00	38.57	24.32

Questions for Quadratic Activity

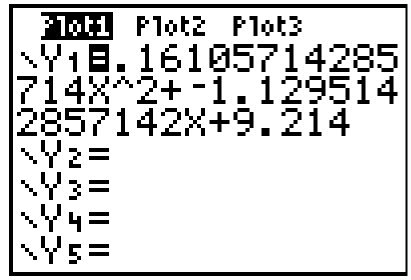
1. Graph the data with the number of students on the x-axis and the time in seconds on the y-axis. Why, in this problem, is it logical to choose the number of students as the independent variable and the time as the dependent variable?



The time it takes to pass the ball depends on the number of students passing the ball.

2. Using a graphing calculator, enter the number of students in one list and the time recorded for each group in a second list. Find an equation that fits the data using the quadratic regression option in the calculator.

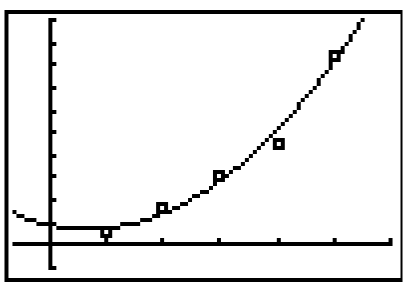
Equation: $y = 0.16\,x^{2} - 1.13\,x + 9.21$



3. Give the domain and range of your equation for this problem situation.

The domain is the set of positive integers and the range is the set of positive real numbers.

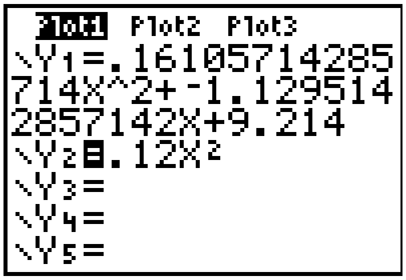
4. Graph the regression equation.

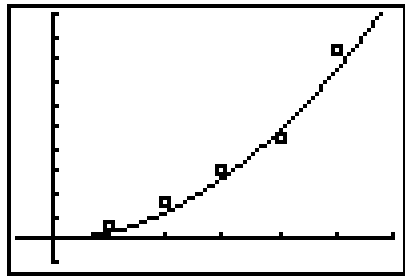


As shown in the data table on the previous page, the second changes in the dependent variable values are not constant. Some students recited the letters of the alphabet at a slower rate than others and some stopped in the middle of their recitation as they became tongue-tied. There was much laughter associated with the data collection as the number of letters increased. If time permits, consider collecting the data a second time (after students practice once) and encourage them to concentrate and try to recite the letters at the same rate as other students in the line.

The graph does appear reasonably parabolic and the quadratic regression equation is a good fit. Note, however, that the quadratic regression equation has a y-intercept of approximately (0, 9.2). Obviously, it does not take over nine seconds for zero people to pass the ball.

When asked to guess the equation, starting with $y = x^2$, the students found the equation $y = 0.12\,x^2$ as shown on the next page. Their y-intercept of (0, 0) makes more sense in this problem situation.





5. Use your regression equation to answer these questions:

a. How long should it take 50 students to pass the ball if they follow the same pattern?

$y = 0.16\,(50)^2 – 1.13\,(50) + 9.21$
$y = 0.16\,(2500) – 56.5 + 9.21$
$y = 352.71$ seconds

It will take approximately five minutes and 53 seconds for 50 students to pass the ball if they follow the same pattern.

b. How many students could pass the ball in 15 minutes if they follow the same pattern.

15 minutes = 900 seconds
$900 = 0.16\,(x)^2 – 1.13\,x + 9.21$

Using the quadratic formula (or the equation solver or Calc (zero) option on a graphing calculator) the positive value of x is 78.23. Approximately 78 students could pass the ball in 15 minutes if they follow the same pattern.

6. If more than 35 (the letter of the alphabet plus the numbers one through nine) students pass the ball following the same pattern, the time required to pass the ball could be affected by the time it takes to say certain numbers. For example, it takes longer to say the number 23 than it does to say the number 5. If you collect and graph data for 100 students, how might that graph compare to the graph of the data collected for your class?

The graph would rise more steeply as it takes more time for larger groups of students to pass the ball.

Questions for Linear Activity

1. Graph the data with the number of students on the x-axis and the time in seconds on the y-axis. Why, in this problem, is it logical to choose the number of students as the independent variable and the time as the dependent variable?

2. If you're using a graphing calculator, press TRACE and move the cursor (using the arrow keys) on the graph to choose two points that you think would be located on a line of best fit. Record the ordered pair for each point. If graphing by hand, pick two points that would be located on a line of best fit.
Points: (,) (,)

3. Find the slope of the line that could be drawn through these two points.

4. Write the slope as a fraction with the appropriate units.

5. What does the slope represent in this problem?

6. Find the equation of a line of best fit. Write the equation of your line of best fit in the form $y = m x + b$. Graph this line.

7. What is the unit for the value of b in your equation?

8. What does the value of b represent in this problem? Note that b is the y- value of the y-intercept of the line.

9. Find the x-intercept. What does the x-intercept represent? Does the x-intercept have a real meaning in this problem?

10. Give the domain and range of your equation for this problem situation.

11. Using your equation, predict the amount of time it would take 100 students to pass the basketball.

12. Using your equation, find the number of students who could pass the basketball in two minutes.

Questions for Quadratic Activity

1. Graph the data with the number of students on the x-axis and the time in seconds on the y-axis. Why, in this problem, is it logical to choose the number of students as the independent variable and the time as the dependent variable?

2. Using a graphing calculator, enter the number of students in one list and the time recorded for each group in a second list. Find an equation that fits the data using the quadratic regression option in the calculator.

Equation: y =

3. Give the domain and range of your equation for this problem situation.

4. Graph the regression equation.

5. Use your regression equation to answer these questions:

a. How long should it take 50 students to pass the ball if they follow the same pattern?

b. How many students could pass the ball in 15 minutes if they follow the same pattern.

6. If more than 35 (the letter of the alphabet plus numbers one through nine) students pass the ball following the same pattern, the time required to pass the ball could be affected by the time it takes to say certain numbers. For example, it takes longer to say the number 23 than it does to say the number 5. If you collect and graph data for 100 students, how might that graph compare to the graph of the data collected for your class?